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D3-branes in NS5-brane backgrounds

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Abstract: We study D3-branes in an NS5-branes background defined by an arbitrary 4d harmonic function. Using a gauge-invariant formulation of Born-Infeld dynamics as well as the supersymmetry condition, we find the general solution for the ω-field. We propose an interpretation in terms of the Myers effect.

Keywords: D-branes, Brane Dynamics in Gauge Theories.

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1. Introduction and summary

Any configuration of parallel NS5-branes creates a non-trivial string background, described by the following fields on the transverse four dimensions :

$$\begin{aligned} G_{\mu\nu} &= V\delta_{\mu\nu} \\ H_{\mu\nu\rho} &= \epsilon_{\mu\nu\rho\sigma}\partial_\sigma V \\ e^{2\Phi} &= V\,. \end{aligned}$$

Here, V can be any harmonic function of the four transverse coordinates X^μ.

These backgrounds play an important rôle in (little) string theory; they are related to various exact string backgrounds. For instance, the near-horizon geometry of k superposed NS5-branes is $\mathbb{R}_\Phi \times \mathrm{SU}(2)$, where $\mathbb{R}_\Phi$ is the linear dilaton background. The near-horizon geometry of NS5-branes spread on a circle [1] is related by T-duality to an orbifold of $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1) \times \mathrm{SU}(2)/\,\mathrm{U}(1)$. In [2], an NS5-branes background is used to exhibit the effect of worldsheet instantons on T-duality. NS5-branes spread on a three-sphere provide another interesting configuration, with a dilaton everywhere finite [3].

All this motivates the study of D-brane probes in such backgrounds. Such probes have already been used in some particular cases of NS5-branes background [4, 5] and in some U-dual configurations [6, 7]. First, the D1-branes (we mean branes extending along one of the four transverse dimensions, with an unspecified number of flat directions parallel to the NS5-branes) are not affected by the NS5-branes and are straight lines.[1] On the contrary, the D3-branes can take quite complicated shapes [5], in relation with the Hanany-Witten effect [8].

In this note we investigate the general properties of D3-branes in *all* such backgrounds. Let us summarize the results. First, an useful tool to study the shapes of D3-branes will be the gauge-invariant rewriting of the Born-Infeld equations of motion, eq. (2.2).

[1] This can be seen from their Born-Infeld action: the V factors coming from the dilaton and metric cancel each other.

We will briefly comment on the geometrical significance of this rewriting in terms of a non-symmetric second fundamental form. Then we will write the Born-Infeld and SUSY equations for D3-branes in general NS5-branes backgrounds. Those two equations turn out to be equivalent. We will find the general solution for the ω-field on the brane, eq. (3.7). This ω-field describes the D1-brane charge of the D3-brane, enabling us to speculate about the D3-branes being formed as bound states of D1-branes via a kind of Myers effect [9].

2. Invariant Born-Infeld equations of motion

The Born-Infeld action reads

$$S_{BI}(X^\mu, F) = \int dx^i L_{BI} = \int dx^i e^{-\Phi} \sqrt{\det\left(\hat{g}_{ij} + \omega_{ij}\right)}\,,$$

where $\omega_{ij} = \hat{B}_{ij} + F_{ij}$ is the gauge-invariant worldvolume two-form, subject to the constraint $d\omega = \hat{H}$ where $dB = H$. The action is gauge-invariant, as well as the equation of motion for the F-field $E^k = -\partial_i \frac{\delta L_{BI}}{\delta F_{ik}}$; but not the equation of motion for the embedding $X^\mu(x^i)$, that is $E_\mu = \frac{\delta L_{BI}}{\delta X^\mu} - \partial_i \frac{\delta L_{BI}}{\delta \partial_i X^\mu}$. However, it is possible to add a combination of E^k to the equation E_μ and to obtain an equivalent, gauge-invariant equation. This was already done in [10], where the equation $E^\mu - E^j B_\nu{}^\mu \partial_j X^\nu = 0$ was used.[2] Here we propose a different combination, which will turn out to have a much more interesting geometrical interpretation :

$$E^\mu + E^j \left(\omega_j{}^k \partial_k X^\mu - B_\nu{}^\mu \partial_j X^\nu\right) = 0\,. \tag{2.1}$$

Indeed this equation may be rewritten

$$-\sqrt{\det\left(\hat{g}+\omega\right)}[(\hat{g}+\omega)^{-1}]^{ji}\left(\partial_i\partial_j X^\mu + \Gamma^\mu_{\nu\rho}\partial_i X^\nu \partial_j X^\rho - \hat{\Gamma}^k_{ij}\partial_k X^\mu\right) - \\ -\sqrt{\det\left(\hat{g}+\omega\right)}\left(\partial^\mu \Phi - \hat{g}^{ij}\partial_i \Phi \partial_j X^\mu\right) = 0\,, \tag{2.2}$$

where we used the spacetime connection

$$\Gamma^\mu_{\nu\rho} = \Gamma(g)^\mu_{\nu\rho} - \frac{1}{2} H^\mu{}_{\nu\rho}\,, \tag{2.3}$$

and the induced worldvolume connection

$$\Gamma^k_{ij} = \Gamma(\hat{g})^k_{ij} - \frac{1}{2}\hat{H}^k{}_{ij}\,. \tag{2.4}$$

The equation (2.2) involves the following two-form with values in the tangent space

$$\Omega^\mu_{ij} = \partial_i\partial_j X^\mu + \Gamma^\mu_{\nu\rho}\partial_i X^\nu \partial_j X^\rho - \hat{\Gamma}^k_{ij}\partial_k X^\mu\,. \tag{2.5}$$

[2] An other way of deriving this equation is to introduce a contravariant worldvolume three-form C^{ijk} in order to impose $d\omega = \hat{H}$, and to consider the action $S'(X^\mu, \omega, C) = \int dx^i e^{-\Phi}\sqrt{\det\left(\hat{g}_{ij}+\omega_{ij}\right)} + \lambda \int dx^i C^{ijk}(d\omega - \hat{H})_{ijk}$. It is then possible to eliminate C^{ijk} from the equations of motion of X^μ, using the equations of ω, now considered as a dynamical field.

This generalizes the second fundamental form and shares its basic properties. Indeed our Ω is transverse ($\Omega^{\mu}_{ij}\partial_k X^{\mu}=0$), and it satisfies generalized Gauss-Codazzi equations

$$R\left(\hat{\Gamma}\right)_{ijkl} = R(\Gamma)_{ijkl} + g_{\mu\nu}\Omega^{\mu}_{[lj}\Omega^{\nu}_{k]i} \tag{2.6}$$

$$(R_N)_{ij}{}^{\mu\nu} = R(\Gamma)^{\mu\nu}{}_{ij} - \hat{g}^{kl}\Omega^{[\mu}_{jk}\Omega^{\nu]}_{il} \tag{2.7}$$

where R_N is the curvature of the spin connection $\omega^{ab}_i = \frac{1}{2}\xi^{[a}_{\mu}(\partial_i + \Gamma^{\mu}_{i\nu})\xi^{\nu b]}$ for some orthonormal basis $\xi^{\mu a}$ of the normal space; explicitly we have

$$(R_N)_{ij}{}^{\mu\nu} = \left(\partial_{[i}\omega^{ab}_{j]} - \omega^{ac}_{[i}\omega^{bc}_{j]}\right)\xi^{\mu}_a\xi^{\nu}_b\,.$$

Thus, we were able to reformulate the Born-Infeld equations of motion in a gauge invariant manner, using the connection with torsion Γ and the associated second fundamental form on the brane. This suggests that those objects should contribute to the derivative corrections to the Born-Infeld action when the B-field is present, generalizing purely gravitational terms of [11, 12]. More generally, this points to the relevance of the connection Γ for the D-branes geometry, as was already noted in [13].

For the moment, we will only use the gauge invariance of eq. (2.2) in order to study D3-branes in an NS5-branes background, without having to fix a gauge for the B-field or to find an F-field on the brane.

3. The case of D3-branes

In order to write the equations which determine the geometry of a D3-brane and its worldvolume two-form ω_{ij}, let us define this geometry by the equation $K(X^{\mu})$ =cst and write the most general local solution to the F-field Born-Infeld equation of motion E^k:

$$\omega_{ij} = V\epsilon_{ijk}\frac{\partial^k\varphi}{\sqrt{1-\hat{g}^{mn}\partial_m\varphi\partial_n\varphi}}\,. \tag{3.1}$$

Here φ is some function on the brane, and we normalize ϵ_{ijk} so that it is a tensor, $\epsilon_{123}=\sqrt{\det\hat{g}}$, where we define $\hat{g}_{ij}=\partial_i X^{\mu}\partial_j X_{\mu}$. We raise spacetime indices with $\delta_{\mu\nu}$, not with the metric $G_{\mu\nu}=V\delta_{\mu\nu}$. Thus, the worldvolume metric $\hat{g}_{ij}$, with which we raise indices, does not coincide with the standard induced metric $V\partial_i X^{\mu}\partial_j X_{\mu}$.

The unknown functions K and φ are subject to two equations, which we write using the projector onto the brane

$$P_{\mu\nu} = \delta_{\mu\nu} - \frac{\partial_\mu K\partial_\nu K}{\partial^\rho K\partial_\rho K}\,.$$

First, we have the gauge-invariant Born-Infeld equation

$$\begin{aligned}\left(P^{\mu\nu} + \frac{P^{\mu\mu'}\partial_{\mu'}\varphi P^{\nu\nu'}\partial_{\nu'}\varphi}{1-P^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi}\right)\partial_\mu\partial_\nu K + V^{-1}\partial_\mu V\partial_\mu K -\\ -\frac{\sqrt{\partial_\alpha K\partial^\alpha K}}{\sqrt{1-P^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi}}P^{\mu\nu}V^{-1}\partial_\mu V\partial_\nu\varphi = 0\,.\end{aligned} \tag{3.2}$$

Second, we should not forget the equation $d\omega_{ij} = \hat{H}$:

$$\left(P^{\mu\nu}+\frac{P^{\mu\mu'}\partial_{\mu'}\varphi P^{\nu\nu'}\partial_{\nu'}\varphi}{1-P^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi}\right)\partial_\mu\partial_\nu K-\frac{\partial_\alpha K\partial^\alpha K}{\partial_\alpha K\partial^\alpha\varphi}\left(P^{\mu\nu}+\frac{P^{\mu\mu'}\partial_{\mu'}\varphi P^{\nu\nu'}\partial_{\nu'}\varphi}{1-P^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi}\right)\partial_\mu\partial_\nu\varphi+ \tag{3.3}$$

$$+\frac{\sqrt{\partial_\alpha K\partial^\alpha K}\sqrt{1-P^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi}}{\partial_\alpha K\partial^\alpha\varphi}V^{-1}\partial_\mu V\partial_\mu K-\frac{\partial_\alpha K\partial^\alpha K}{\partial_\alpha K\partial^\alpha\varphi}P^{\mu\nu}V^{-1}\partial_\mu V\partial_\nu\varphi = 0\,.$$

Our two equations are second-order partial differential equations. It is possible to find a first order equation by studying the supersymmetry condition for the brane, which will turn out to be equivalent to the Born-Infeld equation. First, the background preserves the following supersymmetries :

$$\xi = V^{\frac{1}{16}}\xi_0,\ \xi_0 = \text{ cst}\,,\quad \Gamma_{6789}\xi = -\xi_c\,.$$

Then the D3-brane SUSY condition is

$$-i\sqrt{\det\left(1+\hat{g}^{-1}\omega\right)}\xi = \Gamma_{0\mu\nu\rho}\epsilon^{ijk}\partial_i X^\mu\partial_j X^\nu\partial_k X^\rho\xi + \Gamma_{0\mu}\epsilon^{ijk}\partial_i X^\mu\omega_{jk}\xi_c\,. \tag{3.4}$$

This does not depend at all on the harmonic function V defining the background. With our notations, this can be rewritten in the form $-i\xi = v^\mu\Gamma_{0\mu}\Gamma_{6789}\xi$, with

$$v^\mu = \frac{\sqrt{1-P^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi}}{\sqrt{\partial_\alpha K\partial^\alpha K}}\partial^\mu K + P^{\mu\nu}\partial_\nu\varphi \tag{3.5}$$

At any given point, the existence of such a ξ is guaranteed by the fact that $v_\mu v^\mu = 1$. However, one shoud not forget that ξ should always remain in the same direction, $\xi = V^{\frac{1}{16}}\xi_0$. Thus the SUSY condition for our D3-brane amounts to

$$v^\mu = \text{cst}\,. \tag{3.6}$$

We can use this equation to eliminate φ from our expressions. In particular, the solution for the ω-field is

$$\omega_{ij} = V\frac{\sqrt{\partial_\alpha K\partial^\alpha K}}{\partial_\beta K v^\beta}\epsilon_{ijk}\partial^k X_\mu v^\mu\,,\quad v^\mu\ \text{cst.}. \tag{3.7}$$

The equations (3.2) and (3.3) both take the form

$$\left(P^{\mu\nu}+\frac{\partial_\alpha K\partial^\alpha K}{(v^\beta\partial_\beta K)^2}P^{\mu\mu'}v_{\mu'}P^{\nu\nu'}v_{\nu'}\right)\partial_\mu\partial_\nu K + \left(\partial^\mu K - \frac{\partial_\alpha K\partial^\alpha K}{v^\beta\partial_\beta K}P^{\mu\mu'}v_{\mu'}\right)V^{-1}\partial_\mu V = 0\,.$$

Now that we have studied the local properties of the D3-branes, let us say a word about the quantization conditions. A quantized quantity is, as usual, defined for any two-cycle S^2 of the brane, which is the boundary of some 3-surface M :

$$I = \int_M H - \int_{S^2}\omega\,. \tag{3.8}$$

This quantity measures the RR charge of the D3-brane.

Three situations can happen : first, if the background is created by localized individual NS5-branes, then the quantization is automatically satisfied. Second, if the D3 passes through a stack of NS5, then the angle at which the D3 emerges from this stack is quantized, like in [5]. Third, if the NS5 are spread, then the quantization has to be added by hand. For instance when the NS5 are spread on a circle, then the D3 has to intercept a quantized portion of the circle.

4. Examples and discussion

In this last section we want to give a physical interpretation of our results in terms of the Myers effect. Let us first mention a few examples, where our equations can be solved or at least reduced to differential equations.

- Case when the background is asymptotically flat, $V \to 1$ at infinity : near infinity our D3-brane is nearly flat and we can solve the equations for its shape,

$$K = v_\mu X^\mu \left(V + \frac{\lambda}{r} + O(\frac{1}{r^3}) \right), \tag{4.1}$$

  with $V = 1 + \frac{kl_s^2}{r^2} + O(\frac{1}{r^3})$. The quantized parameter λ measures the number of D1s bound to the flat D3, i.e. the D1-brane charge of the D3, as can be seen by computing the quantity I, eq. (3.8), near infinity.

- $\mathbb{R}_\Phi \times \mathrm{SU}(2)$: the background is defined by $V = \frac{kl_s^2}{r^2}$, and the branes by $K = \frac{v_\mu X^\mu}{r}$ for any constant v_μ. These branes extend along the whole linear dilaton direction, times a standard S^2 conjugacy class in SU(2) [14].

- Superposed NS5-branes: $V = 1 + \frac{kl_s^2}{r^2}$, the partial differential equation on K reduces to a differential equation after assuming $K(v_\mu X^\mu, r)$ (for any constant v_μ). See [5] for more details.

In the case of $\mathbb{R}_\Phi \times \mathrm{SU}(2)$, the S^2 factor of the brane can be formed from a stack of superposed D0-branes on SU(2) via the Myers effect. The $\mathbb{R}_\Phi \times \mathrm{SU}(2)$ background is the near-horizon geometry of the background defined by superposed NS5-branes, and in this case the D3 should be considered as a bound state of a flat D3 with a stack of D1-branes ending on the NS5s (see figure 1). This is confirmed by computing its D1-brane charge. However, this only holds when the flat D3-brane does not go too far from the NS5s, so that the throat of the corresponding curved D3 is not as thin as the string length ℓ_s. A limiting case occurs when we consider D1-branes going to infinity without ending on any D3; then there is no D3-brane solution which would be a candidate for a bound state of those D1s, and the Myers effect does not occur. This might seem a bit strange when we look at the near-horizon limit, but one should not forget that this region suffers from a strong coupling problem and cannot be expected to give reliable information. On the contrary, in the flat region far from the NS5s, the D1s are not expected to form any bound state.

Now, we are led to conjecture that this phenomenon is general and that every supersymmetric D3-brane in an NS5-branes background is a bound state of D1s (with or without a flat D3-brane at infinity depending on the background). The main evidence we have is the existence of the constant vector v^μ, which indicates the direction of the D1-branes of interest. Our D3 preserves the same supersymmetries as those D1-branes and should have the correct charge, as hinted by eq. (3.7). It would be interesting to study this kind of Myers effect using the nonabelian Born-Infeld action, however one should take into account the fact that the original D1-branes may end on a D3-brane.

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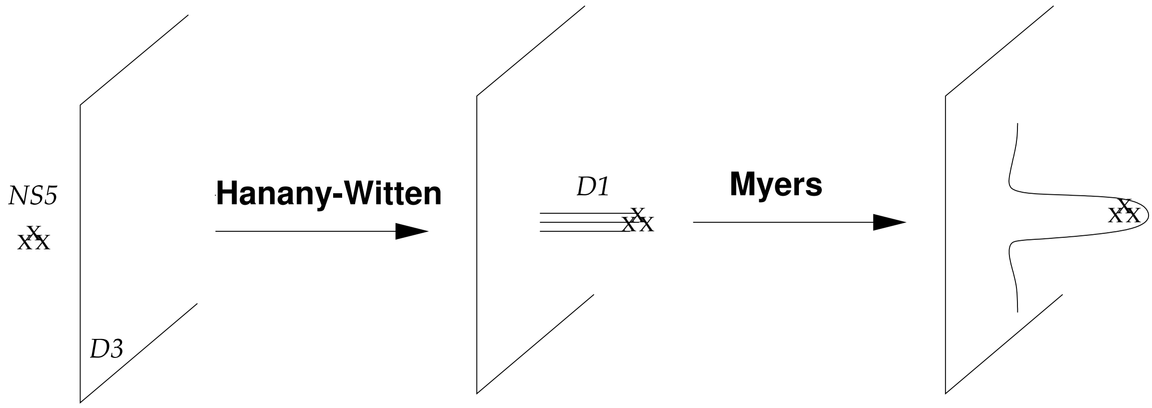

Figure 1: A case of the proposed Myers effect.

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